# Fermi National Accelerator Laboratory

Cosmological Lower Bound on

Heavy Neutrino Masses

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## ABSTRACT

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of $2 \times 10^{-29}$ g/cm$^3$, the lepton mass would have to be <u>greater</u> than a lower bound of the order of 2 GeV.

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There is a well-known cosmological argument[1] against the existence of neutrino masses greater than about 40 eV. In the "standard" big-bang cosmology,[2] the present number density of each kind of neutrino is expected[3] to be 6/11 the number density of photons in the 3°K black-body background radiation, or about 300 cm$^{-3}$; hence if the neutrino mass were above 40 eV, their mass density would be greater than $2 \times 10^{-29}$ gm/cm$^3$, which is roughly the upper limit allowed by present estimates[4] of the Hubble constant and the deceleration parameter.

However, this argument would not apply if the neutrino mass were much larger than 1 MeV. Neutrinos are generally expected[2] to go out of thermal equilibrium when the temperature drops to about $10^{10}$°K, the temperature at which neutrino collision rates become comparable to the expansion rate of the universe. If neutrinos were much heavier than 1 MeV, then they would already be much rarer than photons at the time when they go out of thermal equilibrium, and hence their number density would now be much less than 300 cm$^{-3}$.

Of course, the familiar electronic and muonic neutrinos are known to be lighter than 1 MeV. However, heavier stable neutral leptons could easily have escaped detection, and are even required in some gauge models.[5] In this article, we suppose that there exists a neutral lepton $L^0$ (the "heavy neutrino") with mass well above 1 MeV, and we assume that $L^0$ carries some additive or multiplicative quantum number which keeps it absolutely stable. We will present arguments based on the standard big-bang cosmology to show that the mass of

such a particle must be above a lower bound of order 2 GeV.

At first glance, it might be thought that the present number density of heavy neutrinos would simply be less than the above estimate of 300 $cm^{-3}$ by the value $\exp(-m_L/1 \text{ MeV})$ of the Boltzmann factor at the time the heavy neutrinos go out of thermal equilibrium. If this were the case, then an upper limit of $2\times10^{-29} g/cm^{-3}$ on the present cosmic mass density would require that $m_L \exp(-m_L/1 \text{ MeV})$ should be less than 40 eV, and hence that $m_L$ should either be less than 40 eV or greater than 13 MeV.

However, the true lower bound on the heavy neutrino mass is considerably more stringent. The heavy neutrinos are assumed to carry a conserved quantum number, so their number density can relax to its equilibrium value $n_0$ only by annihilation of a heavy neutrino with a heavy antineutrino, in processes such as

$$L^0\bar{L}^0 \rightarrow \nu\bar{\nu}, \ e^-e^+, \ \mu^-\mu^+, \ \pi^-\pi^+, \text{ etc.} \tag{1}$$

But although the energy distribution of the heavy neutrinos is kept thermalized by collisions with $\nu,e$,etc. down to a temperature of $10^{10} \, {}^\circ K$, at that temperature they are so rare that their annihilation rate is already much less than the cosmic expansion rate. Thus the heavy neutrinos go out of chemical equilibrium, (in the sense that their number density begins to exceed its equilibrium value) at a "freezing" temperature $T_f$ which is much higher than 1 MeV. The condition on $m_L$ is then that $m_L \exp(-m_L/kT_f$ should be less than 40 eV, and the resulting lower bound on $m_L$ must therefore be greater than 13 MeV.

To make this quantitative, we use the rate equation

$$\frac{dn}{dt} = - \frac{3\dot{R}}{R} n - <\sigma v>n^2 + <\sigma v>n_0^2 \quad . \tag{2}$$

Here  n  is the actual number density of heavy neutrinos at time t; R is the cosmic scale factor; $<\sigma v>$ is the average value of the $L^0\bar{L}^0$ annihilation cross-section times the relative velocity and $n_0$ is the nu ber density of heavy neutrinos in thermal (and chemical) equilibrium[6]:

$$n_0(T) = \frac{2}{(2\pi)^3} \int_0^\infty 4\pi p^2 dp \left[\exp\left((m_L^2+p^2)^{\frac{1}{2}}/kT\right) + 1\right]^{-1} \quad . \tag{3}$$

(We use units with $\hbar=c=1$ throughout.)

At the temperatures we are considering here, the energy density and the entropy are dominated by highly relativistic particles, including  $\gamma,\nu_e,\nu_\mu$, and e.  It follows that R, T and t are related by

$$\dot{R}/R = -\dot{T}/T = (8\pi\rho G/3)^{\frac{1}{2}} \tag{4}$$

where $\rho$ is the energy density

$$\rho = N_F a T^4 = N_F \pi^2 (kT)^4/15 \tag{5}$$

with $N_F$ an effective number of degrees of freedom, counting $\frac{1}{2}$ and 7/16 respectively for each boson or fermion species and spin state.  For temperatures in the range of 10-100 MeV (which most concern us here) we must include just $\gamma,\nu_e,\bar{\nu}_e,\nu_\mu,\bar{\nu}_\mu,e^-$, and $e^+$, so $N_F = 4.5$, a value we will adopt for most purposes.  However, if current ideas about the strong interactions are correct, then $N_F$  rises steeply at a temperature of order 500 MeV to a value[7] $N_F \approx 30$.

To estimate $<\sigma v>$, we note that the heavy neutrinos must be quite non-relativistic at the temperature $T_f$ where they freeze

out of chemical equilibrium, because as we have seen, the upper

limit on the present cosmic mass density requires the Boltzmann

factor $\exp(-m_L/kT_f)$ to be very small. For non-relativistic

velocities, the cross-section $\sigma(v)$ for the exothermic processes

(1) behaves like $1/v$, so we can take $<\sigma v>$ as a temperature-

independent constant. If there were only a single annihilation

channel open (say $L^0\bar{L}^0 \rightarrow \nu\bar{\nu}$) and if the annihilation process was

due to an ordinary "V minus A" charged-current Fermi interaction,

then $<\sigma v>$ would be just $G_F{}^2 m_L{}^2/2\pi$, where $G_F$ is the Fermi coupling

constant $1.15\times10^{-5}\text{GeV}^{-2}$. To take account of more general

possibilities, we shall write

$$<\sigma v> = G_F{}^2 m_L{}^2 N_A/2\pi \qquad (6)$$

where $N_A$ is a dimensionless fudge factor which depends both on

the number of annihilation channels open and on the details of

the weak $L^0\bar{L}^0$ annihilation interaction. (In general, the

annihilation proceeds both through exchange of charged and of

neutral intermediate vector bosons.) Where a numerical value is

needed, we will take $N_A=14$. [This corresponds to the annihilation

channels $L^0\bar{L}^0 \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau, e^-e^+, \mu^-\mu^+, u\bar{u}, d\bar{d}, s\bar{s}$, with all

fermions (antifermions) of helicity $-\frac{1}{2}$ $(+\frac{1}{2})$, and three colors for

each of the quarks u,d,s.] New channels open up for $m_L$ above about

5 GeV, leading to a moderate rise in $N_A$.

Equations (3)-(6) allow us to rewrite the rate equation (2)

in the convenient form

$$df/dx = C\mu^3 (f^2 - f_0{}^2) \qquad (7)$$

where

$$x = kT/m_L \quad , \qquad \mu^3 = m_L{}^3 N_A \Big/ \sqrt{N_T} \quad ,$$

$$f(x;\mu) = n/T^3 \quad , \tag{8}$$

$$f_0(x) = n_0/T^3 = \pi^{-2} k^3 \int_0^\infty du\, u^2 [\exp(x^{-2}+u^2)^{\frac{1}{2}}+1]^{-1},$$

and

$$C = \frac{1}{k^3} G_F{}^2 \left(\frac{45}{32\pi^5 G}\right)^{\frac{1}{2}} = 1.28\times10^6 \left(\frac{cm\,^\circ K}{GeV}\right)^3 \quad . \tag{9}$$

Equation (7) is to be solved subject to the initial condition that as $x\to\infty$, $f(x)$ approaches the equilibrium value $f_0(x)$. For $x\ll1$, $f(x)$ approaches an asymptotic value, which evidently depends only on the single unknown parameter $\mu$:

$$\lim_{x\to0} f(x;\mu) = F(\mu) \quad . \tag{10}$$

The subsequent annihilation of electron-positron pairs increases RT by a factor $(11/4)^{1/3}$, and hence decreases $n/T^3$ by a factor 4/11, so the present mass density of heavy neutrinos (and antineutrinos) will be given by

$$\rho_L = 2\left(\frac{4}{11}\right) m_L T_\gamma{}^3 F(\mu) \tag{11}$$

with $T_\gamma = 3\,^\circ K$ the present radiation temperature. (For freezing temperature above 100 MeV the factor 4/11 must be decreased to take account of the subsequent heating of photons by annihilation of $\mu^-\mu^+, \pi^-\pi^+$, etc.).

Computer solutions of Eq. (7) are shown for a variety of special cases in Fig. 1. We find that over the range of parameter considered here, the function F of Eq. (10) can be very well represented by

$$F(\mu) \simeq 1.20 \times 10^{-5} (cm^\circ K)^{-3} [\mu (GeV)]^{-2.85} \quad . \tag{12}$$

The present mass density of heavy neutrinos and antineutrinos is then given by Eq. (11) as

$$\rho_L = (4.2 \times 10^{-28} g/cm^3) \times [m_L (GeV)]^{-1.85} (N_A/\sqrt{N_F})^{-0.95} \quad . \tag{13}$$

If we require[4] that $\rho_L < 2 \times 10^{-29} g/cm^3$, then $m_L$ must be subject to the lower bound

$$m_L (N_A/\sqrt{N_F})^{0.51} \gtrsim 5.2 \text{ GeV} \quad . \tag{14}$$

For $N_A=14$, $N_F=4.5$, this gives $m_L \gtrsim 2$ GeV. Allowing for a factor of 4 uncertainty in the $L^0 \bar{L}^0$ annihilation rate, the lower bound on $m_L$ would lie in the range 1-4 GeV.

It is enlightening to see how these results can be obtained by an analytic approximation to the asymptotic value of $n/T^3$. We expect that $f(x)$ remains approximately equal to $f_0(x)$ until the temperature drops to a freezing value $T_f$, at which the annihilation rate per unit volume equals the rate of change of $n_0$, or

$$df_0/dx = C\mu^3 f_0^2 \quad , \quad \text{at } x = x_f = kT_f/m_L \quad , \tag{15}$$

and that thereafter $f$ obeys approximately the equation

$$df/dx = C\mu^3 f^2 \quad , \quad x < x_f \quad , \tag{16}$$

with the initial condition $f(x_f) = f_0(x_f)$. Since $kT_f \ll m_L$, we can approximate $f_0$ in Eq. (8) by the non-relativistic formula

$$f_0(x) = \frac{n_0}{T^3} \simeq 2k^3 (2\pi x)^{-3/2} e^{-1/x} \tag{17}$$

and take only the exponential into account in evaluating $df_0/dx$. The freezing temperature is then obtained from

[6]We suppose here that the chemical potentials associated with any conserved quantum numbers carried by $L^0$ are vanishingly small. If there were any appreciable degeneracy, the mass density of the heavy neutrinos would greatly exceed reasonable cosmological limits.

[7]This includes contributions of $\gamma, \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, e^-, e^+, \mu^-, \mu^+$; plus three color triplets u,d,s of quarks and the corresponding antiquarks; plus eight massless spin-one gluons.

FIGURE CAPTION

Fig. 1    $n/T^3$ vs. T for a variety of special cases of $m_L$, $N_F$ and $N_A$.
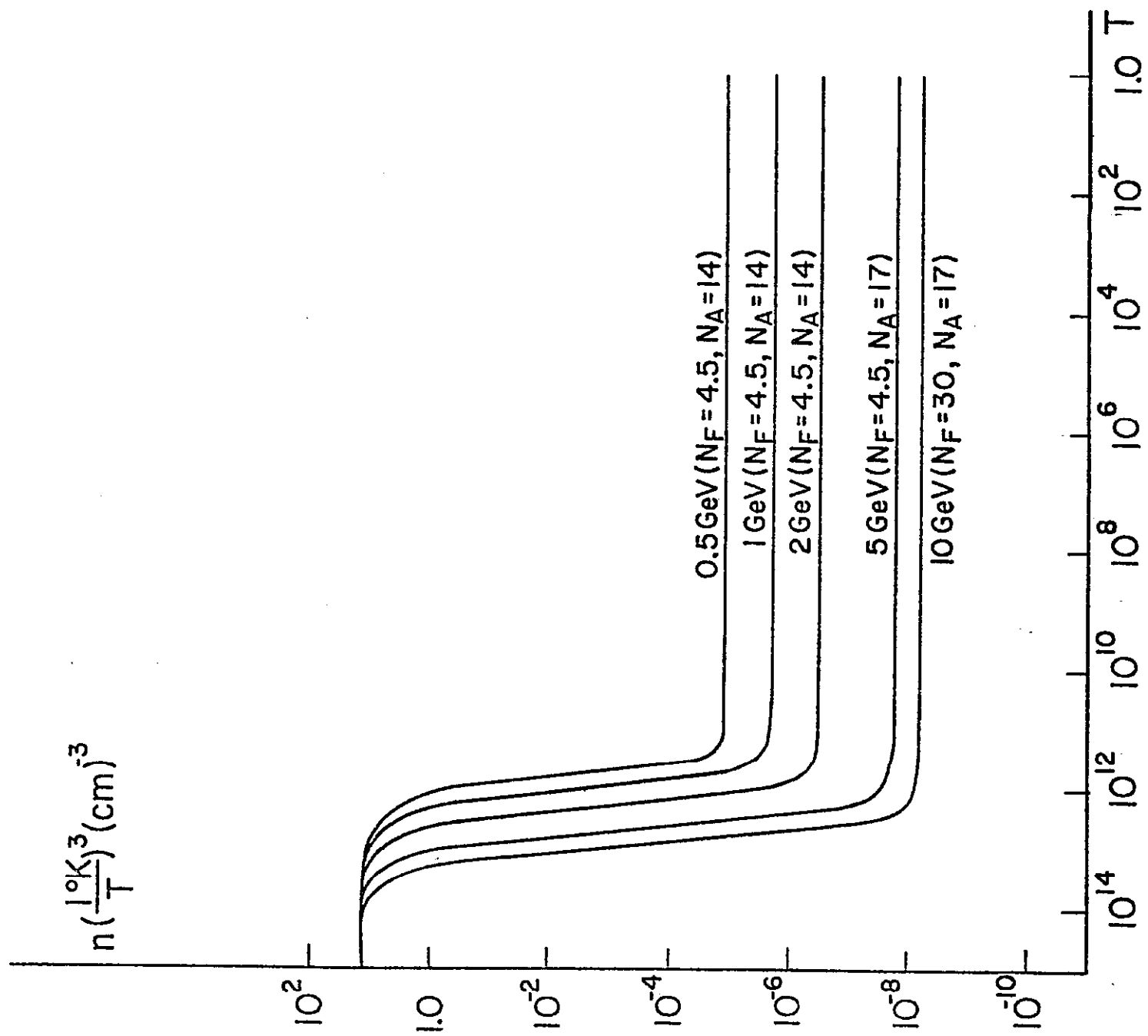
$$n\left(\frac{1^\circ K}{T}\right)^3 (cm)^{-3}$$

0.5 GeV ($N_F$=4.5, $N_A$=14)

1 GeV ($N_F$=4.5, $N_A$=14)

2 GeV ($N_F$=4.5, $N_A$=14)

5 GeV ($N_F$=4.5, $N_A$=17)

10 GeV ($N_F$=30, $N_A$=17)

T

FIG. 1